

THE SIX-PHASE GENERATOR AND TRANSFORMER INTO
A THREE-PHASE POWER SYSTEM

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Abstract - A model six-phase generator has been tested together with model transformers giving six-to three-phase conversion with a view to assessing the harmonic performance of the system. The measured harmonic currents are shown to be consistent with those calculated from the equivalent circuits of the generator and transformer. It is shown that voltage harmonics of order $12k \pm 1$ are passed into the power system but those of order $12k \pm 5$ can only produce circulating current within the generator and transformer when the h.v. output windings are put in parallel. Because the leakage reactances of large transformers are about 15% even at fundamental frequency it is not thought that harmonic effects would seriously compromise performance.

INTRODUCTION

The growth in output of the largest two-pole generators has given rise to a variety of problems. Recent growth has been achieved with very little increase in overall size but substantially by increase in the stator current density. The stator current per slot is now so large that the forces tending to push the bars to the bottom of the slots at twice supply frequency are regrettably large. Projected increases in output of two-pole machines to 1300 MVA make it necessary to consider methods by which the current per slot and the corresponding forces on the stator bars may be reduced. Two methods have been suggested. In a two-pole machine only two perfectly balanced parallel circuits are available in each phase. More parallel circuits can be obtained by mixing the phase bands, but this complicates the winding connections and the voltage balance is only approximate. The alternative is to increase the number of phases. Machines with two three-phase windings were made in U.S.A. in the 1930's to reduce fault levels, the two windings having the same phase. The suggestion made more recently, is for each phase winding to be spread over 30° and for the windings to be connected to give two three-phase outputs which are 30° apart in phase.^{1,2,3} The generator is connected to the 3-phase power system by a star/delta transformer on one winding and a star-star transformer on the other. With this arrangement the stator winding factor is improved and 6% more output can be produced.⁴ Addition losses are studied in ref.5

Multi-phase machines have been suggested for use with inverter drives for variable speed use in order to suppress space harmonics. More significantly for

present purposes, multi-phase transmission systems appear to be in active consideration.^{6,7} Thus this study of a six-phase generator and the use of transformers with star and delta primaries to halve the number of phases has a relevance not foreseen when this work was started.

A 3kVA micro-machine specially built by Mawdsleys Ltd., has been tested together with transformers made within Imperial College. The immediate aim of the work has been to establish the steady state performance with particular regard to the generation of harmonic voltages within the machine, and the danger of harmonic circulating currents between the two three-phase windings through paths provided within the transformer, or being fed into the system. The equivalent circuits have been obtained and the factors limiting the circulating current are given. Under normal circumstances with due care the circulating currents can be kept small.

The Generator

The generator has twelve phase belts per pole-pair which may be connected to give six or three-phase output as shown in Figure 1. The "micro" machine particulars are given in table I and although the machine has 4 poles and its windings are not similar to those of a large machine, it is electrically similar on a parameter pu basis.

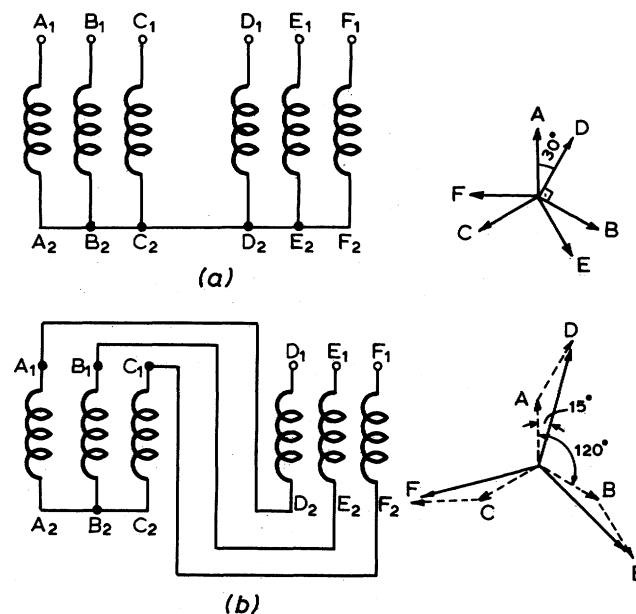


Fig.1. Stator winding connection in six phases (a) and 3 phases (b) showing the voltage phasors.

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TABLE I

Particulars of model stator
Winding connected in 6 phases.

Phase voltage	62 V
Full load phase current	8.0 A
Number of poles	4
Frequency	50 Hz
Number of slots	48
Number of conductors per slot	12
Number of turns per phase	48
Conductor size (copper)	2 (10 mm. x 1.25 mm.)
Type of winding	2-tier, double layer
Coil pitch	10/12
Winding factor	.955
Length of mean turn	40" (102 cm.)
Phase resistance, 15 deg. C	.035Ω
Slot skew	1 slot pitch
Axial length, gross	5.5" (14 cm.)
Axial length, effective	5.2" (13.2 cm.)
Air gap	1.27 mm.

The open circuit voltage has harmonic components arising from the harmonic components of the rotor field and from saliency. The stator voltage at each frequency also depends on the appropriate winding factors of the stator winding. These are readily calculable for a 30° spread of conductors, i.e. for a phase group in the six-phase machine. In the three phase machine two such phase groups are connected in series and the winding factor is decreased by their phasor difference. Thus for the nth harmonic voltage,

$$\frac{U_{n6}}{U_{n3}} = \frac{1}{2} \frac{1}{\cos 15n} \quad (1)$$

1/cos15 = 1.035, but for the 5th and 7th harmonics 1/cos15n ≈ 4, so these harmonics are a greater fraction of the fundamental in the six-phase machine, and it is from these that the danger of circulating current principally exists.

The mmf's produced by stator current in the six-phase machine are also greater because of the higher winding factors. However the pattern of space harmonics is different, only those of order h = 12k±1 being generated in the six phase machine, compared with h = 6k±1 in the three phase machine. Thus in the six-phase machine 5th and 7th harmonic space mmf are not produced by fundamental current and the higher winding factors, being small, are immaterial. Table II shows the gap fields of space harmonic h produced by stator current of harmonic number n, their motion being indicated

relative to the fundamental field. For each harmonic of current a space harmonic is produced which is stationary with respect to the fundamental field and the rotor when n = h. This field is defined here as that corresponding with the magnetising reactance at that value of n. The remaining air-gap fields moving with respect to the rotor may be regarded as leakage components. This definition corresponds with the normal definition of unsaturated magnetising reactance for fundamental current given by:

$$X_{m1} = \frac{4m}{\pi} f \mu_0 \frac{TL}{pcg} k^2 \omega_1 N_{ph}^2 \Omega/ph \quad (2)$$

For the nth harmonic magnetising reactance is less.

$$X_{mn} = \frac{X_{m1}}{h} \left(\frac{k_{\omega n}}{k_{\omega 1}} \right)^2 \quad (3)$$

This approach to harmonic magnetising reactance ignores the rotor saliency as is appropriate to stator generated harmonic fields. A more rigorous solution would require a full consideration of the rotor and stator shape, the degree of saturation and the currents, probably in a field solution.

The generator leakage reactance is different from that in a three-phase machine because the conductors in adjacent phase belts carry current 30h° different in phase rather than 60h°. Components of leakage are slot, end, zig-zag and belt as defined by Alger⁹ and Liwshitz⁸ for fundamental current but extended here to give values for harmonic currents.

The slot leakage component for a double-layer winding from each slot, l_s is given by the inductance of each layer l_A, l_B and the mutual inductance between them,

$$l_s = l_A + l_B + 2k_r l_{AB} \quad (4)$$

where k_r allows for the phase difference. For the whole winding k_r is given by

$$k_r = \frac{1}{2q} \sum_1^{2q} \cos \alpha \quad (5)$$

Maintaining Alger's nomenclature⁹

$$k_s = \frac{1 + k_r}{2} \quad (6)$$

and then for fundamental current

$$X_{slot} = 2\pi f s (l_A + l_B + 2l_{AB} (2k_s - 1)) \quad (7)$$

and for harmonic n

$$X_{slot,n} = 2\pi n f s (l_A + l_B + 2l_{AB} (2k_{sn} - 1)) \quad (8)$$

TABLE II

Speed and rotation of space harmonics of stator mmf in three and six phase windings. Brackets indicate presence only in the three phase connection, synchronous speed is n = 1 and h = 1

Order of space harmonic h	Order of time harmonic, n				
	1	5	7	11	13
1	+1	(- 5)	(+ 7)	- 11	+ 13
3					
5	(- 1/5)	+ 1	- 7/5	(+ 11/5)	(- 13/5)
7	(+ 1/7)	- 5/7	+ 1	(- 11/7)	(+ 13/7)
9					
11	- 1/11	(+ 5/11)	(- 7/11)	+ 1	- 13/11
13	+ 1/13	(- 5/13)	(+ 7/13)	- 11/13	+ 1

where k_{sn} is given by the curves shown in Figure 2 for both the three and six phase connection. It will be seen that:

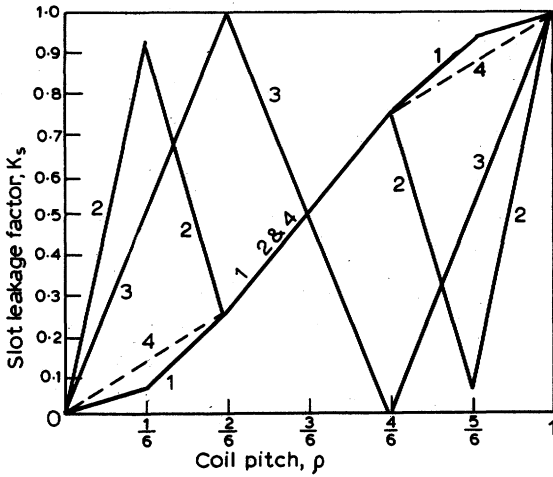


Fig.2. Slot leakage factor, k_s , for 3 and 6 phase machines with 60° and 30° phase belts

- 1. Harmonic currents of orders $12k \pm 1$ } 6ph
- 2. Harmonic currents of orders $12k \pm 5$ }
- 3. Harmonic currents of orders $6k \pm 3$ }
- 4. Harmonic currents of orders $6k \pm 1$ } 3ph

1. In the three-phase machine the variation of k_s with winding pitch is the same for current harmonics $n = 12k \pm 1$ or $12k \pm 5$. Consequently slot leakage reactance with respect to these harmonics is n times that for fundamental current. In the six-phase machine this relationship does not hold except when $1/3 < \rho < 2/3$.
2. Slot leakage reactance for $n = 12k \pm 5$ in the six-phase machine is much less than that for the three-phase machine when pitch is $5/6$. With this coil pitch the lower value of leakage reactance favours the flow of circ-

ulating current of this order $n = 5, 7, 17, 19 \dots$. Even with pitch of $11/12$ the harmonic current might be greater than in a three phase machine

3. The slot leakage factor for triplen harmonics currents is the same for three and six-phase machines.

Slot leakage factor k_s is independent of the number of slots per pole.

End and zig-zag leakage components are relatively low and may be obtained by Alger's formulae⁹. Belt leakage corresponding with air gap harmonic fields travelling with the rotor have been neglected as they will be greatly reduced by rotor damping effects.

The magnetising and leakage reactances for the model machine are given in table III.

Generator Tests

Attempts to measure the small harmonic voltages of the generator on open circuit were unsuccessful, the rejection capability of the wave analyser used being inadequate. Values of current for a short across all the phases (Fig.3 with the switch closed) were calculated as

$$I_{an} = \frac{U_n}{X_{Ln} + X_{mn}} \tag{9}$$

belt leakage being neglected, and are compared with measured values in table IV for the machine connected in three and six phases. Values agree reasonably closely ($\pm 10\%$) up to the seventh harmonic but for $n = 11$ and 13 for the six-phase connection agreement is poor. The reason for this discrepancy is not understood but as the values fluctuated from one instant to the next it is thought the measurement may be at fault, the signals being very small. Current waveforms are shown in Fig.4. Third harmonic current flows in this condition from one three-phase set to the other, as their induced voltages

TABLE III

Magnetising and leakage reactance for current harmonic n in Ω/ph

n	Magnetising reactance		slot leakage reactance		end leakage reactance		zig-zag leakage		Total leakage reactance	
	3Ph	6Ph	3Ph	6Ph	3Ph	6Ph	3Ph	6Ph	3Ph	6Ph
1	29.2	15.7	1.12	.6	.244	.131	.154	.0822	1.52	.812
3	3.28	1.64	2.0	1.0	-	-	-	-	2.0	1.0
5	.0168	.125	5.62	.324	.0064	.0476	.0308	.0012	5.66	.373
7	.0061	.046	7.87	.5	.004	.03	.022	.0008	7.9	.531
11	.022	.0117	12.37	6.6	.0027	.00146	.014	.0075	12.4	6.59
13	.0133	.0071	14.62	7.8	.00267	.0014	.018	.0063	14.6	7.77

TABLE IV

Measured (m) and computed (c) short-circuit harmonic currents in the micro-machine connected in 6 (and 3 phases with a line-neutral connection below) phases at different excitation currents. Values in A.

Field Current I_f	I_{a_1}		I_{a_3}		I_{a_5}		I_{a_7}		$I_{a_{11}}$		$I_{a_{13}}$	
	m	c	m	c	m	c	m	c	m	c	m	c
1	4.5	4.66	.6	.66	.56	.59	.5	.54	.03	.009	.022	.0085
	4.63	4.8	.6	.66	.026	.027	.02	.0203	.008	.0095	.01	.0087
1.2	5.4	5.6	.7	.79	.66	.71	.6	.65	.036	.011	.026	.01
	5.6	5.76	.7	.79	.32	.032	.024	.24	.01	.011	.012	.01
1.4	6.2	6.5	.81	.924	.75	.82	.7	.76	.042	.013	.031	.012
	6.45	6.72	.81	.92	.036	.038	.027	.028	.011	.013	.014	.012
1.6	7.1	7.5	.92	1.05	.86	.944	.79	.865	.049	.014	.035	.014
	7.4	7.68	.92	1.1	.043	.043	.032	.0325	.012	.015	.017	.014
1.8	8	8.4	1.05	1.2	.96	1.06	.89	.97	.055	.016	.04	.015
	8.3	8.64	1.04	1.2	.047	.048	.036	.365	.014	.017	.02	.0156
2	8.9	9.32	1.15	1.32	1.1	1.18	.99	1.08	.06	.018	.044	.017
	9.3	9.6	1.15	1.32	.053	.054	.04	.041	.016	.019	.022	.0173

are 90° different in phase. The current in one group (ABC) is in antiphase to that in the other (DEF). The magnitude is identical to that which flows if the machine is connected with 3 phases and a line-neutral connection. Third harmonic currents can be avoided in a short circuit test by using two separate shorting links, one for each phase group, i.e. the switch in Fig.3 open.

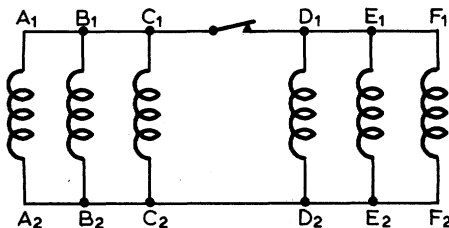


Fig.3. 6-phase generator in short-circuit

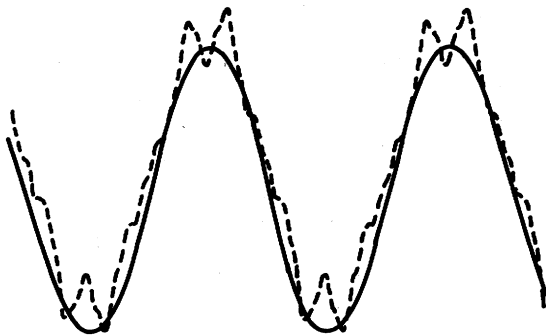


Fig.4. Short-circuit phase current without L-N connection
 ——— 3ph
 - - - 6ph

The Transformer Arrangement

At 1300 MVA it would almost certainly be essential to use 3 bank transformers and it is this approach that has been modelled here. The height of such units would be restricted by transport limitations and a four-limb construction was adopted as shown in Fig.5. Each inner limb had a low voltage primary, a high voltage secondary and a tertiary winding. In order to simulate the high p.u. leakage reactance of a full-size transformer (15%), and the small p.u. copper loss, a large air gap was left between the primary and secondary windings and a heavy conductor was used. A leakage reactance of 9% and a copper loss of 0.6% was obtained. Higher values of leakage reactance and the same copper loss would have been difficult to obtain at a reasonable cost. In constructing the model transformers great care was taken in making the joints between the laminations for their reluctance is a high fraction of the total when compared with their contribution in a full-size transformer. However the magnetising current was still large, about three times the appropriate p.u. value for a large transformer. At abnormally high flux densities the effect of the joints becomes less significant. The three transformers were designed for use as shown in Fig.6. To obtain fundamental current balance the p.u. leakage reactance between each l.v. winding and its secondary must be kept the same although the delta-connected windings require a factor of $\sqrt{3}$ more turns than the primary winding connected in star. The balance in leakage reactance is obtained by ensuring that each primary winding occupies the same window area and has the same position relative to each secondary winding.¹⁰ The high voltage secondary windings occurring in two sections per phase may be connected in series as shown in Fig.6, or put in parallel. The latter requires a lower level of insulation on one winding than when they are in series and would probably be cheaper.

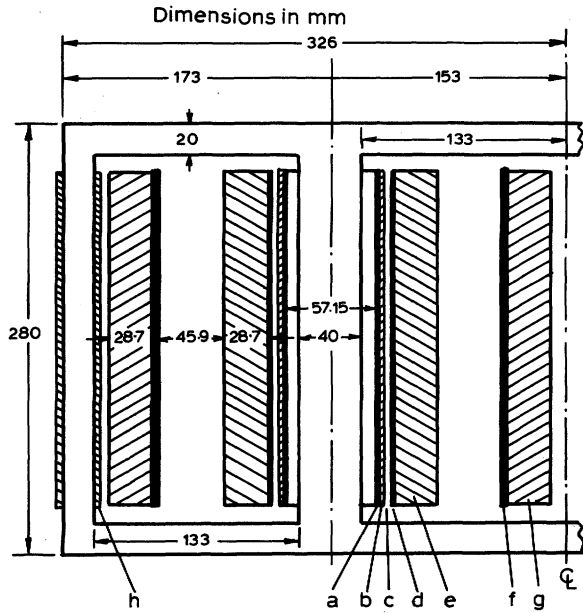


Fig. 5. Model transformer design. The core is 40 mm deep.

- a. base cylinder
- b. tertiary winding
- c. spacing
- d. base cylinder
- e. l.v. winding
- f. base cylinder
- g. h.v. winding
- h. search coil

The generator transformers may be represented by equivalent circuits and these have been obtained by methods given by Carpenter.¹¹ The actual magnetic and electric arrangement of Fig. 7a is replaced by the equivalent magnetic circuit with currents in the meshes of Fig. 7b. These mesh sources of m.m.f. are replaced by equivalent branch sources in Fig. 7c, half-transformers representing the linkages with the electric circuits being inserted wherever the magnetic circuit is cut by removal of the mesh currents. The dual of this circuit,

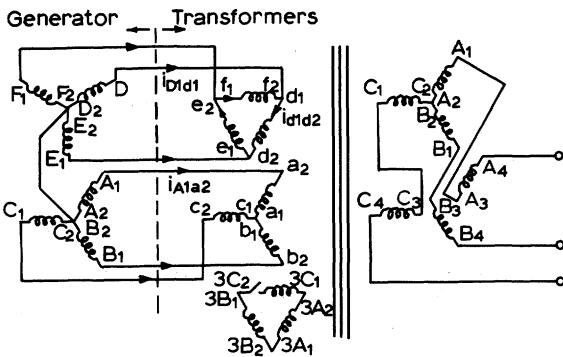
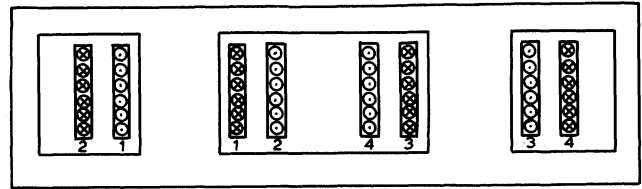
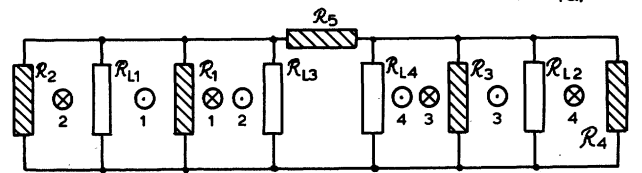


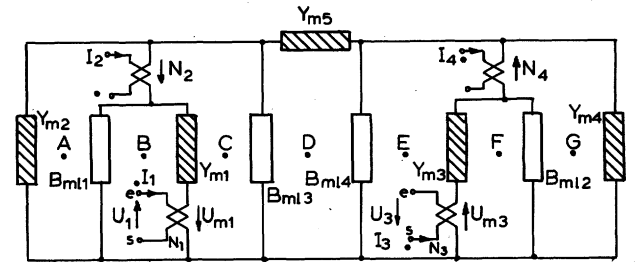
Fig. 6. 6-phase generator and its three 6/3ph transformers. Transformer windings on one core are shown parallel with each other. h.v. windings are shown in series.



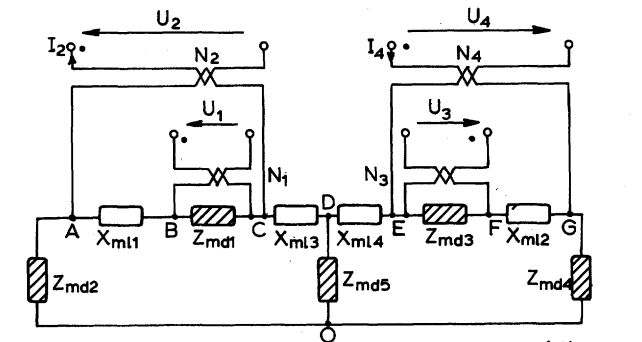
(a)



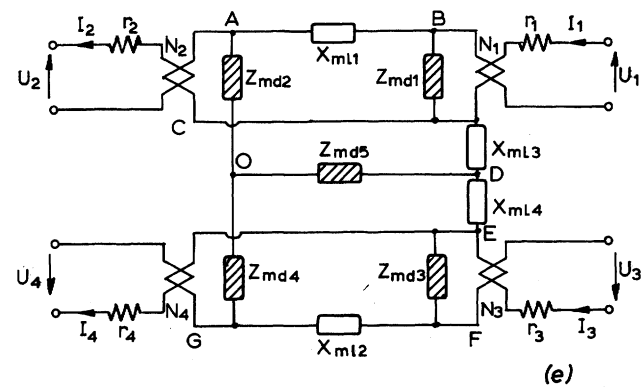
(b)



(c)



(d)



(e)

Fig. 7. Development of equivalent circuit for four-winding transformer.

- a. Transformer with coils in section
- b. Discrete model with iron reluctances (shaded) and air reluctances
- c. As (b) with half-transformers representing mmf.
- d. Dual of (c)
- e. (d) rearranged and with circuit resistances added.

Fig.7d, gives the equivalent magnetic circuit in terms of magnetising reactances and four lossless air leakage reactances. This circuit is rearranged in Fig.7e to include the resistances of the windings. The advantage of this approach is that circuit impedances retain a direct connection with the elements they represent.

With the secondaries on open-circuit each transformer has the equivalent circuit of Fig.8a. Thus if $V_1 \neq V_3$ the impedance faced by circulating current corresponds with the flux in the air-leakage paths and that in the outside limbs. If the outside limbs were omitted the reactance of the total air leakage would remain.

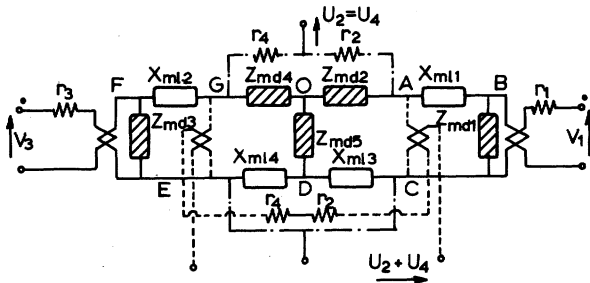


Fig.8. Equivalent circuits for four-winding transformer

- a. ——— Secondary windings open
- b. - - - - Secondary windings in parallel (added to (a)).
- c. - - - - Secondary windings in series (added to (a)).

When the secondary (h.v) windings are put in parallel with each other, Fig.8b, the impedance between the driving voltages is reduced to the sum of the reactances representing the air leakage flux between the windings in each limb and the resistance of the secondary windings.

If the secondaries are put in series, Fig.8c, the impedance between the primary voltages remains similar to that on open circuit.

The generator and transformer

The nth time harmonic voltage across the l.v. windings of the transformer can be described by

$$U_{a2-a1,n} = U_n \sin(n(\omega t - 120q) + \alpha_n) \tag{10}$$

for the star connected windings where q = 0,1 and 2 for phases A,B and C, and

$$U_{d2-d,n} = U_n 2\sin(60n)\sin(90n)\sin(n(\omega t - 120q) + \alpha_n) \tag{11}$$

for the delta-connected winding where q = 0, 1 and 2 for phases D,E and F. The magnitude of fundamental voltage per turn is made equal by the $\sqrt{3}$ greater number of turns in the delta-connected windings but the phases of the harmonic voltages are different and are shown in Fig.9. This information is summarised in Fig.10. This shows that:

- (i) For harmonics of order $n = 12k \pm 1$ including the fundamental the voltages in the primaries are in phase and these cannot be circulating current between them unless the magnitudes are not equal.
- (ii) For harmonics of order $n = 12k \pm 5$ the harmonic voltages are in antiphase in each transformer and it is for $n = 5$ and 7 that the greatest danger of circulating current occurs.

- (iii) Harmonics of order $n = 6k + 3$ (triplens) would only occur in the generator output if both star points were connected and there would then be voltage between the two three-phase sets. These are avoidable but the

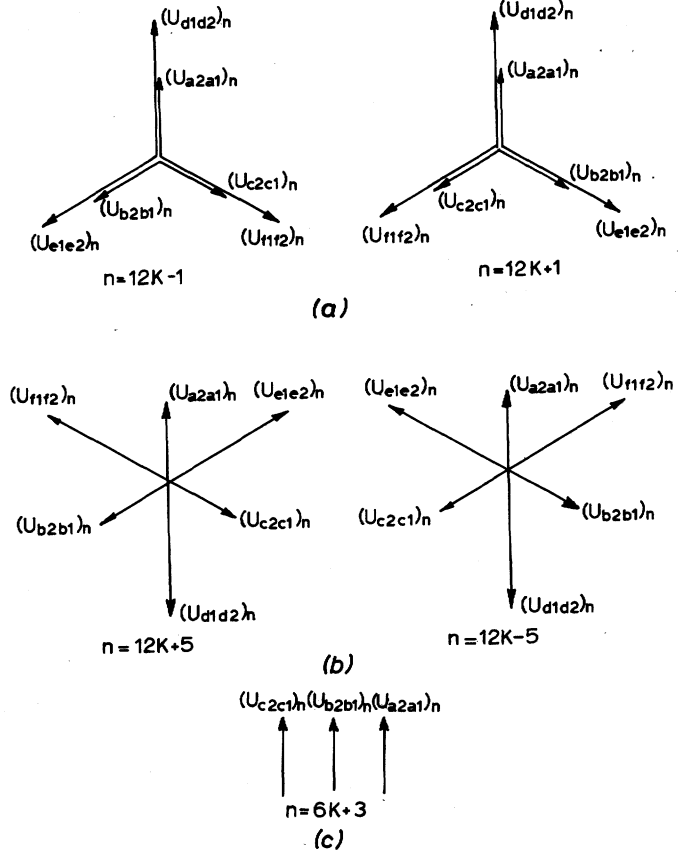


Fig.9. Relative phases of voltage harmonics applied to the transformer l.v. windings

third harmonic effects of sinusoidal voltage excitation on the transformer are not, the saturating magnetising characteristic, requires third harmonic current for fundamental flux. In the delta-connected winding third harmonic currents circulate freely, but as these are not possible in the star-connected winding voltage distortion occurs. The voltage waveform is improved by use of the tertiary winding as in any other multi-phase transformer.

Examination of the equivalent circuits shows that whatever harmonic voltage of order $n = 12k \pm 1$ is in the l.v.windings will also be present in the output windings when on open circuit or when connected in series or in parallel.

For $n = 12k \pm 5$ no voltage appears in the output. The harmonic voltage in the h.v. winding sections are of opposite phase so that when the sections are in series this voltage sums to zero. When the sections are in parallel they assist each other to produce circulating current in the h.v. windings. With the secondaries open flux is put into the outer limbs and air leakage paths by both primary windings producing mmf in phase (upwards at the same instant) as in Fig.10b. In the absence of the outer limbs the impedance would increase to that corresponding with the air leakage paths between the two primary windings which is still substantial.

Of all these conditions the least impedance to circulating current is presented when the output windings are in parallel. The limiting leakage reactance is then

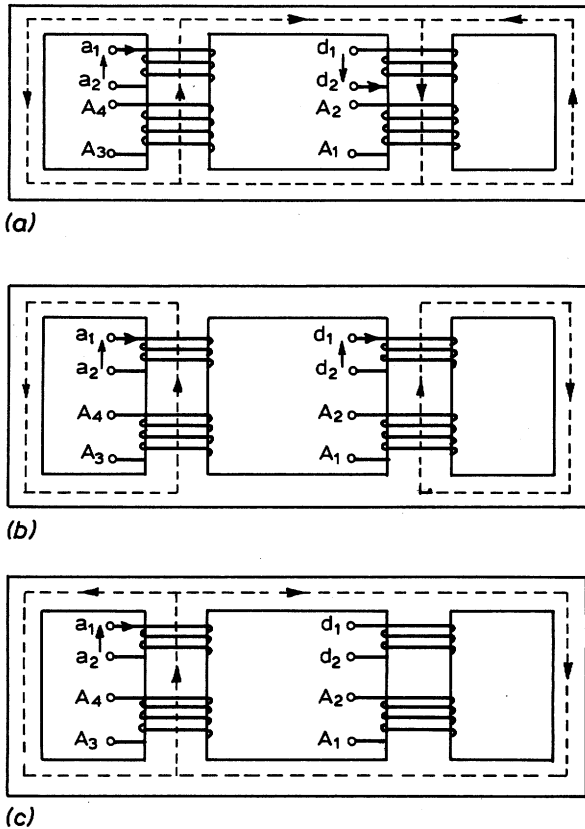


Fig. 10. Voltage phase and flux paths for n th harmonic
 a). $n = 12k+1$, b). $n = 12k+5$, c). $n = 6k+3$

merely that between the $l.v.$ and $h.v.$ windings on each limb.

The suppression of the $n = 12k+5$ ($n = 5, 7, \dots$) harmonics in the transformer output is the corollary of the suppression of the space harmonics of this order in the stator mmf. The presence of $n = 12k+1$ ($n = 11, 13, \dots$) in the output is similar to that which would occur if the generator were used as a 3-phase machine. Magnitudes are slightly increased in the 6-phase arrangement by 30° shift between the two outputs.

Generator-Transformer tests on load

Fig. 11 shows transformer $h.v.$ winding current waveforms on load and table V shows the harmonic content. With the $h.v.$ windings in parallel it was found that harmonic current in each winding increased with field current but there was virtually no harmonic current in the output lines. With no tertiary winding the paralleled $h.v.$ windings carried third harmonic current. When the tertiary was employed this current became much smaller. Table V indicates the presence of 5th and 7th harmonics in the winding sections ($n = 12k+5$) as predicted. The only harmonics possible in the transformer output are of order $n = 12k+1$ (11, 13, ...). These are small being faced with the high leakage reactance corresponding with their frequency.

With transformer $h.v.$ windings in series and the tertiary windings connected, both primary and secondary currents of the transformer were almost sinusoidal. In this condition the voltages of order $n = 12k+1$ add in the $h.v.$ windings, but produce negligible current.

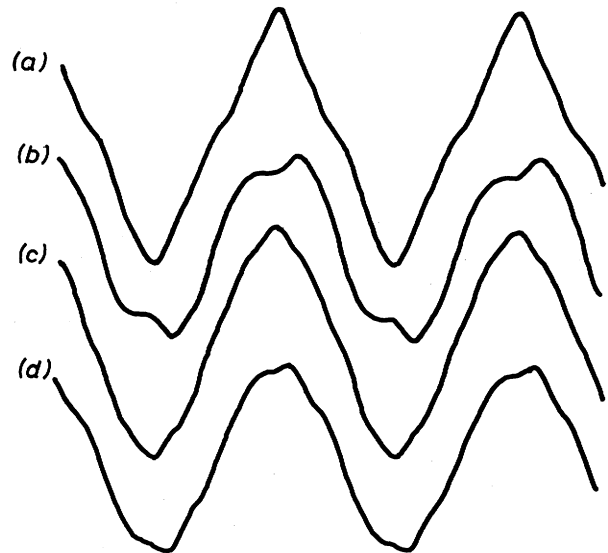


Fig. 11. Current in each transformer $h.v.$ winding (load and circulating) when supplying half-rated load at 0.8 pf (lagging) with $h.v.$ windings in parallel.

- a. $h.v.$ winding with primary in star } tertiary open
- b. $h.v.$ winding with primary in delta } tertiary open
- c. as (a) } tertiary closed
- d. as (b) } tertiary closed

TABLE V

Harmonic current magnitudes corresponding with Fig. 11

	Time Harmonic n			
	1	3	5	7
a	3.04	.42	.056	.082
b	2.86	.38	.08	.056
c	3	.074	.062	.078
d	2.8	.02	.086	.054

CONCLUSIONS

Generator 5th and 7th current harmonics are minimized by the use of a coil pitch of 11/12.

Triplen harmonic currents can be allowed to flow in the transformer delta and the $h.v.$ windings of the transformer if these are in parallel. Alternatively the use of a tertiary winding removes this current from the windings carrying load current.

The flow of harmonic currents of order $n = 12k+1$ arising from the harmonic mmf of the generator rotor depends on the connection of the transformer $h.v.$ windings. When they are in series the current is small because it has to flow out into the system and the series impedance appears to be large. If a large bank of capacitors were near the generator the current flow might require more careful consideration. However, the transformer $h.v.$ windings are much more likely to be connected in parallel as this does not require the additional winding insulation on one set of secondary windings necessary with windings in series. In this

condition these harmonics circulate between the h.v. windings. Figure 8b shows the transformer equivalent circuit in this condition and the summing voltages in windings 1 and 3 are faced with the sum of the leakage reactances between windings on the two wound limbs in each phase. Fortunately these are substantial, 15% for fundamental in a large unit. Four-limb construction of each transformer has no effect on the magnitude of this order of harmonic when the h.v. windings are in parallel. When they are in series harmonic flux is pushed into the outer limbs. In their absence it would cause additional loss in leakage paths such as the tank.

Further details are available in Ref.12 and alternative transformers for this duty are discussed in Ref.10.

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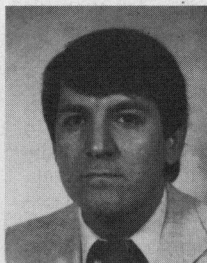
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Symbols

B	Susceptance Ω^{-1}
C	allowance in the air gap for slot openings, >1
f	frequency Hz
g	air gap m
h	integer of space harmonic
I_{an}	phase current, neutral to phase A
k	integer
k_r	leakage factor for slot with 2 phases
k_s	see equation 6
kw	winding factor
L	machine effective length, m
m	no. of phases
n	integer of time harmonic
N_{ph}	turns/phase
p	no. of pole pairs
q	slots/pole/phase
R	reluctance AT/Weber
r	resistance - Ω
U_n	voltage of harmonic n
s	slots/phase
X_{Ln}	leakage reactance for harmonic n
X_{mn}	magnetising reactance for time harmonic n. Ω
Y	admittance Ω^{-1}
Z	impedance Ω
α	phase difference between currents in the same slot
α_n	stator electrical slot angle for harmonic n
τ	pole pitch m
μ_0	$4\pi \times 10^{-7}$ H/m
ω	angular frequency rad/s
ρ	fractional coil pitch, $0 > \rho > 1$



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